Uncertainty Quantification in Trajectory Prediction for Aircraft Operations

Shankar Sankararaman*
SGT Inc., NASA Ames Research Center, Moffett Field, CA 94035, USA

Matthew Daigle†
NASA Ames Research Center, Moffett Field, CA 94035, USA

This paper presents a computational methodology for uncertainty quantification in predicting the trajectory of a generic, realistic aircraft based on information regarding flight plan, aircraft information, wind and weather information, etc. Predicting the trajectory of aircraft is important from the point of view of analyzing and predicting the safety of the overall airspace, and making risk-informed decisions regarding the operations of the airspace. The proposed methodology is based on using first-principles for analyzing the motion of the aircraft and estimating its future trajectory. Since the core of this problem lies in predicting the future behavior of a generic aircraft, it is essential to understand that it is almost impossible to precisely predict the future trajectory with certainty. Hence, an intuitive approach is to analyze the various sources of uncertainty that affect the aircraft prediction and quantify their combined effect on the whole trajectory. Further, this paper implements a global sensitivity analysis-based methodology to quantify the relative contributions of the various sources of uncertainty to the uncertainty of the overall trajectory. The proposed methodology is illustrated using a numerical example consisting of an aircraft that takes off from the San Francisco International Airport.

I. Introduction

The National Airspace System (NAS) is continuing to evolve with new operational paradigms (e.g., dynamic Traffic Flow Management, Trajectory-Based Operations) and additional aircraft (e.g., Unmanned Aerial Systems). In this increasingly complex system, maintaining safety becomes increasingly challenging. NAS upgrades in the form of the FAA's Next Generation Air Transportation System (NextGen) aims to change the way the NAS operates such that capacity is expanded while safety is still ensured.[1]

It is very clear that knowledge regarding the trajectory of an aircraft plays an undeniably important role in assessing the safety of not only that particular aircraft but also the entire airspace as a whole. As a result, several researchers have studied trajectory prediction,[2–5] planning[6–8] and optimization[9–12] for aircraft in the context of NAS.

Some of the aforementioned papers have pursued probabilistic/statistical approaches for analyzing and predicting aircraft trajectories. This is mainly because of the fact that there are several sources of uncertainty that affect the operations and the overall safety of the NAS, and such uncertainties are best handled using probabilistic methods. In fact, some of the above papers do agree that it is important to understand the impact of such sources of uncertainty on the NAS and aircraft operations, and quantify the effect of these uncertainties on aircraft trajectories.

Uncertainties affect not only aircraft trajectories but also the NAS as a whole. It can be argued[13,14] that it is important to analyze and quantify the various sources of uncertainty that affect the NAS, systematically quantify their impact on the operations of the NAS, estimate the effect of uncertainty on safety, and aid risk-informed decision-making activities to ensure smooth operation of the overall National Airspace System.

*Research Engineer, NASA Ames Research Center (SGT Inc.), Moffett Field, CA 94035, AIAA Senior Member
†Research Computer Scientist, NASA Ames Research Center, Moffett Field, CA 94035, AIAA Member

As a result, there are a few researchers who have studied the impact of uncertainty on NAS operations and overall safety. For example, Landry and Archer[15] have focused on enumerating the different sources of uncertainty in the NAS. In this work, several sources of uncertainty were identified; however, the relationships between them and their overall effects on NAS-level safety were not addressed. In several research manuscripts,[16–18] flow models for air traffic management have been discussed and these models are able to account for some types of uncertainty. These uncertainties are mostly modeled using simplistic Gaussian variables that may rely on unrealistic assumptions; the true underlying distribution may not be Gaussian and needs to be estimated, and such cases have not been addressed. Further, this analysis has been started only for a few uncertain variables (say, for instance, departure delays), and several other sources of uncertainty have not been studied, modeled, or accounted for. Weather has been accounted for in a few publications[19] and the methods discussed are mostly based on collecting existing data and using simplistic linear regression-based models. The effects of weather delays in the overall NAS (system-wide, not localized to a specific area) and delay propagation have not been studied in detail. Only a few sources of uncertainty have been researched in some detail. For example, Wanke et al.[20] discuss quantifying uncertainty in airspace demand predictions, Kim et al. [21] and Garcio-Chico et al.[22] discuss trajectory uncertainty modeling, Tu et al.[23] study estimating flight departure delays. As acknowledged by several of the aforementioned papers, research in the context of uncertainty analysis appears to be at a very early stage in general and further research is still needed to improve the state-of-the-art. There are several problems that need to be addressed in the context of the NAS.

This paper takes a further step in this direction by focusing on uncertainty in aircraft trajectories, and developing a computational framework for efficiently quantifying the uncertainty in aircraft trajectories. In order to accomplish this, it is first necessary to identify/enumerate the various sources of uncertainty that affect aircraft trajectory, and then systematically quantify the combined effect of such uncertainties on the overall aircraft trajectory. This is in stark contrast with many existing approaches that assign/compute error bounds for trajectories after the analysis required for trajectory prediction; the proposed approach advocates the systematic inclusion of uncertainty right from the beginning, and quantifying the effect of uncertainty on the aircraft trajectory using mathematically sound approaches. While quantifying uncertainty is only one side of the coin, other interesting questions are: "What uncertainties are more important than others? How much does each of the individual uncertainties contribute to the overall uncertainty in an aircraft trajectory?" Such issues have never been addressed. In an effort towards answering such questions, this paper also investigates the application of Sobol's indices[24] and global sensivity analysis[25] methods for this purpose.

The rest of this paper is organized as follows. Section II presents the computational approach for trajectory prediction and modeling. Then, Section III discusses the impact of uncertainty on trajectory prediction and investigates methods for uncertainty quantification. Then, Section IV presents the methodology for global sensitivity analysis, using which it is possible to examine which sources of uncertainty are significant from the point of view of trajectory prediction. Numerical results are presented in Section V, and finally, conclusions and future work are discussed in Section VI.

II. Trajectory Prediction and Modeling

The NAS is made up of many interacting components: aircraft, weather systems, pilots, controllers, etc. For the purpose of predicting aircraft trajectory, this paper considers only the open-loop case, where aircraft are operating independently of each other and controllers are not interfering with their intended flight paths. For the purposes of this paper, we consider only one aircraft and its flight plan along with a predicted trajectory.

A. Modeling Framework

To begin with, a general state-space framework for aircraft trajectory prediction is presented. The first step is to develop a model describing the dynamics of the aircraft, i.e., how the state $\mathbf{x}$ evolves in time:

$$\mathbf{x}(k+1) = \mathbf{f}(k, \mathbf{x}(k), \mathbf{u}(k), \mathbf{v}(k)), \tag{1}$$

where $\mathbf{f}$ is the state function, $\mathbf{u}$ is the input vector (exogenous inputs to the system, such as the aircraft's intended flight routes and wind velocity at various altitudes), and $\mathbf{v}$ is the process noise vector. The state equation allows us to compute future values of the state given the inputs and the state at the time of prediction. Note that t represents continuous-time and k represents discrete-time (while the models below are presented in terms of continuous time, implementation is in terms of discrete time).

In order to make a prediction at time k using $\mathbf{f}$, we require $\mathbf{x}(k)$, which, in general, is not known. Instead, we have available an output vector $\mathbf{y}$, defined through an output equation:

$$\mathbf{y}(k) = \mathbf{h}(\mathbf{x}(k), \mathbf{u}(k), \mathbf{n}(k)), \tag{2}$$

where $\mathbf{h}$ is the output function, and $\mathbf{n}$ is the sensor noise vector.

B. Trajectory Definition

The trajectory and the state-space models need to be defined such that the output of the above state-space modeling framework would correspond to the aircraft trajectory. Consider a generic time t_P at which the trajectory needs to be predicted; then the trajectory consists of a vector of quantities that need to be predicted for all time $t_P < t < t_H$, where t_H represents the time-horizon, i.e., a future time-instant until which the trajectory needs to be predicted.

These quantities, as mentioned before, correspond to the outputs $\mathbf{y}(t)$, and include:

$$\mathbf{y}(t) = \begin{bmatrix} V_g(t) \\ \chi_g(t) \\ h(t) \\ \lambda(t) \\ \tau(t) \end{bmatrix}, \tag{3}$$

where V_g is the groundspeed, χ_g is the aircraft heading (based on the groundspeed vector), h is the mean sea level (MSL) altitude, λ is the latitude, and τ is the longitude. The heading χ_a is defined clockwise from the north. If these quantities are predicted for future times $t_P < t < t_H$, then it is equivalent to predicting the trajectory.

C. Dynamic Models

This section discusses the dynamic models that are used in this paper for trajectory prediction. Note that the aforementioned state-space framework can be used with any set of dynamic models for aircraft trajectory prediction. While high-fidelity simulations[11] may be available, simpler models are recommended in this paper in order to facilitate uncertainty analysis in real-time aircraft trajectory prediction and safety analysis.

This paper uses kinematic models of aircraft navigation with simplified dynamics and control, similar to the models developed by others.[26–28] The following description presents differential equations in continuous time t; for implementation purposes, they are converted to difference equations using a sampling time of 10 s. The aircraft state vector is defined as

$$\mathbf{x}(t) = \begin{bmatrix} V_a(t) \\ V_z(t) \\ \chi_a(t) \\ h(t) \\ \lambda(t) \\ \tau(t) \end{bmatrix}, \tag{4}$$

where V_z is the climb rate, V_a is the indicated airspeed, and χ_a is the aircraft heading based on the airspeed vector.

The input vector is defined as

$$\mathbf{u}(t) = \begin{bmatrix} V_a^*(t) \\ V_z^*(t) \\ \chi_a^*(t) \\ V_w(t) \\ \chi_w(t) \end{bmatrix}, \tag{5}$$

where V_a^* is the commanded airspeed, V_h^* is the commanded climb rate, χ_a^* is the commanded aircraft heading, V_w is the wind speed, and χ_w is the wind heading.

The latitude and longitude dynamics are given by

$$\dot{\lambda} = \frac{V_a \sin \chi_a + W_N}{R_e + h}, \tag{6}$$

$$\dot{\tau} = \frac{V_a \cos \chi_a + W_E)}{(R_e + h) \cos \lambda}, \tag{7}$$

where R_e is the MSL radius of the Earth, W_N is the northern component of the wind vector, and W_E is the eastern component of the wind vector:

$$W_N = V_w \cos \chi_w, \tag{8}$$

$$W_E = V_w \sin \chi_w. \tag{9}$$

For the speed and headings, simple dynamics are assumed, and the aircraft moves to its commanded values with some inertia:

$$\dot{h} = V_z, \tag{10}$$

$$\dot{V}_z = \frac{1}{J_z}(V_z^* - V_z), \tag{11}$$

$$\dot{V}_a = \frac{1}{J_a}(V_a^* - V_a), \tag{12}$$

$$\dot{\chi}_a = \frac{1}{J_\chi}(\chi_a^* - \chi_a), \tag{13}$$

where the J terms are the inertia parameters.

The groundspeed and ground-relative heading are computed from the vector addition of the airspeed vector (magnitude V_a and heading χ_a) and the wind vector (magnitude V_w and heading χ_w). The remaining trajectory variables are directly those in the state vector (h, λ, and τ).

In order to simulate ahead, the inputs $\mathbf{u}$ must be defined for the prediction interval $[t_P, t_H]$. The inputs are computed as follows. First, the phase of flight is determined (takeoff, en-route, landing). For takeoff, the comanded heading is set to the end of the runway, the commanded airspeed set to the predetermined takeoff speed, and the climb speed set at 0 until a fast enough groundspeed is acheived, at which it is set to a predetermined climb speed (which is uncertain). In en-route, the aircraft is commanded to follow the flight plan waypoints at cruise speed and cruise altitude. In landing, the commanded airspeed is set to the predetermined landing speed, the commanded heading pointing towards the runway, and the commanded descent speed set to a predetermined value.

Using the above models, it is possible to enumrate all quantities (though not exhaustively since the models are only an approximation of the underlying reality) that affect aircraft trajectory prediction; some of these quantities are uncertain and their effect on aircraft trajectory is discussed in the next section.

III. Uncertainty Quantification in Aircraft Trajectory

The topic of uncertainty quantification has received significant attention and researchers have developed both probabilistic[29–31] and non-probabilistic[32–34] methods for this purpose; non-probabilistic approaches have rarely been considered in air traffic analysis and hence, are not discussed further in this paper. Nevertheless, a systematic approach to quantifying uncertainty in aircraft trajectory has still not been developed in detail. As mentioned before, existing works either focus only on a very small subset of uncertainties and mostly perform a posteriori analysis of uncertainties through computing errors bounds. On the contrary, this paper advocates the inclusion of uncertainties right from the start of the prediction analysis and mathematically computes the effect of such uncertainties on aircraft trajectory prediction. The impact of uncertainty on generic applications involving future predicton has been studied to some extent,[35] and this paper extends the investigation for trajectory prediction.

Typically, there are three steps involved in uncertainty analysis for a generic problem involving prediction:

1. **Uncertainty Identification and Quantification:** This involves enumerating and computing the statistics of all quantities that affect the quantity of interest, i.e., aircraft trajectory in the context of this paper. In other words, these are the sources of uncertainty that affect trajectory prediction. This includes quantities such as wind speed/direction, initial position of the aircraft, departure time, etc. In this step, probability distributions for each of these quantities are computed. Note that it may not be reasonable to assume that these distributions are Gaussian; they may be of any type or even non-parametric.[36]

2. **Uncertainty Propagation:** This involves the use of computational tools to systematically compute the joint effect of the aforementioned sources of uncertainty and quantify the probability distribution of the entire trajectory as a whole. This can be accomplished by estimating the joint effect of the various sources of uncertainty on the quantities in Eq. 3, and quantifying the probability distributions of these quantities for all future time-instants, i.e., $t_P < t < t_H$.

3. **Uncertainty Management:** Uncertainty management is a general term used to refer to different activities which aid in managing/reducing uncertainty in the final prediction. There are several aspects of uncertainty management. For instance: 'What if the uncertainty in the aircraft trajectory is too high? Is it possible to reduce/improve the uncertainty estimates?' The answers to these questions lie in identifying which sources of uncertainty are significant contributors to the uncertainty in the trajectory prediction. Some aspects of this process will be discussed later in Section IV.

A. Sources of Uncertainty

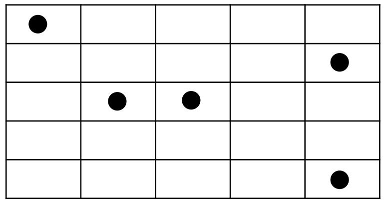
The following sources of uncertainty are considered in this paper:

1. Initial air speed ($V_a(t)$) at the time of prediction (t_P)
2. Initial climb speed ($V_z(t)$) at the time of prediction (t_P)
3. Initial heading ($\chi_a(t)$) at the time of prediction (t_P)
4. Initial altitude ($h(t)$) at the time of prediction (t_P)
5. Initial latitude ($\lambda(t)$) at the time of prediction (t_P)
6. Initial longtitude ($\tau(t)$) at the time of prediction (t_P)
7. Future wind speed (V_w) for all time $t_P < t < t_H$
8. Future wind heading (χ_w) for all time $t_P < t < t_H$
9. Commanded climb speed (V_h^*) during the takeoff portion of the flight
10. Commanded airspeed (V_a^*) during the takeoff portion of the flight
11. Control parameter (G_z), affecting how fast the aircraft responds to commanded changes in climb speed
12. Control parameter (G_a), affecting how fast the aircraft responds to commanded changes in airspeed
13. Model inertia parameter (J_χ)

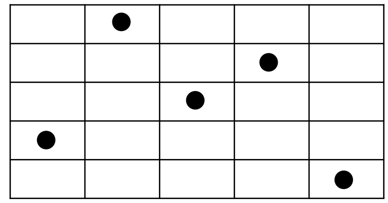
B. Trajectory Prediction: An Uncertainty Propagation Problem

For a given realization of all the above quantities, there exists a unique trajectory, i.e., a value of $\mathbf{y}(k)$ in Eq. 2, for all time $t_P < t < t_H$. Let $\boldsymbol{X}$ denote the vector of quantities enumerated in earlier in Section A. Let Y denote any of the quantities that correspond to the trajectory at any particular time instant, as mentioned earlier in Eq. 3.

There exists a realization of Y corresponding to a realization of $\boldsymbol{X}$. In general, this relation can be expressed as a function $Y = G(\boldsymbol{X})$. The goal is to propagate all the uncertainty in $\boldsymbol{X}$ (expressed in terms of the probability distribution of $\boldsymbol{X}$) through G, and quantify the probability distribution of Y.



(a) Random 2-D Sampling



(b) 2-D Latin Hypercube Sampling

Figure 1: Illustrating Random 2-D Sampling and 2-D Latin Hypercube Sampling

C. Methods for Uncertainty Propagation

Monte Carlo sampling[37] is a well-known approach in order to perform uncertainty propagation. A standard Monte Carlo approach generates many realizations of $\boldsymbol{X}$, and evaluates G for each realization. Thus, many realizations of Y can be computed, and hence, the probability distribution of Y can be computed.

However, Monte Carlo sampling may require using a large number of samples, and hence an equal number of evaluations of G. In particular, exhaustive sampling may be necessary to accurately capture information regarding the tails of probability distributions where unsafe events are captured (since they have lower likelihood of occurrence). Hence, in many scenarios, such exhaustive Monte Carlo sampling could be cost-prohibitive or not suitable for real-time operations.

In order to overcome this challenge, this paper uses Latin Hypercube Sampling that focuses on generating samples consistently over the entire domain of all random variables. Consider N_d random variables, and say, it is desired to generate N_s samples. First, the range of each variable is divided into N_s equally probable intervals, thereby forming a hypercube. In order to facilitate such division into equally probable intervals, the original probability distributions of the variables are not considered at first, and instead, the samples are first generated from the standard uniform random variable [0, 1]. Then, sample positions are chosen such that there is exactly one sample in each row and exactly one sample in each column of this grid (similar to placing rooks on a chess board so that no rook may attack another). The resulting hypercube can be denoted as U_{ij} where 'i' denotes the sample number (varies from 1 to N_d) while 'j' denotes the random-variable number (varies from 1 to N_d). The difference between two-dimensional random Monte Carlo sampling and two-dimensional Latin hypercube sampling is illustrated in Fig. 1a (Monte Carlo samples) and Fig. 1b (Latin Hypercube samples).

Each generated Latin Hypercube sample is then passed through the function G, and corresponding samples of Y are generated, thus generating samples for the entire trajectory. This set of samples establishes a probability distribution (in terms of density) for the trajectory-related variables. An additional advantage of the Latin hypercube sampling approach is that it is a variance reduction technique. Traditional Monte Carlo sampling is highly non-deterministic and may produce significantly different results when repeated (especially when the number of samples is small). On the other hand, Latin hypercube sampling reduces such variation effectively. This in turn further improves the prediction of probabilities and event times in the context of safety assessment.[13]

IV. Global Sensitivity Analysis for Aircraft Trajectory

The goal in sensitivity analysis is to apportion the uncertainty in Y to the uncertainty in inputs $\boldsymbol{X}$. The topic of sensitivity analysis is closely associated with uncertainty propagation, and Saltelli et al.[25] state that uncertainty propagation is incomplete without the results of quantitative sensitivity analysis. Saltelli et al.[25] explain that derivative-based local sensitivities are not sufficient to study the contributions of multiple sources of uncertainty to the overall prediction uncertainty and it is necessary to pursue a global sensitivity analysis approach for this purpose. The term "global" refers to computing the sensitivity metric considering the entire probability distribution of the input.

The fundamental theorem that governs the development of the global sensitivity analysis methodology is the variance decomposition theorem. Consider a particular input quantity X^i. Then,

$$V(Y) = V(E(Y|X^i)) + E(V(Y|X^i)) \tag{14}$$

The above variance decomposition is true if and only if there exists a value of Y for every value $\boldsymbol{X}$; in other words, G is a deterministic transfer function, as explained at the beginning of this section.

In order to compute the sensitivity of a particular input quantity X^i, this input quantity is first fixed at a particular deterministic value and the expectation of the model output is calculated by considering the variation in other output quantities (denoted by X^{-i}). Thereby, the effect of the uncertainty of all other input quantities is averaged. Then, different deterministic values of the input quantity X^i are considered based on their probability distributions and the variance of the expectation is calculated. This metric is known as the first-order effect index of the input variable X^i on the variance of the output Y:

$$S_1^i = \frac{V_{X^i}(E_{X^{-i}}(Y|X^i))}{V(Y)} \tag{15}$$

The first-order effect measures the contribution of the variable X^i by itself. The sum of first order indices of all variables is always less than or equal to unity. The difference between this sum and unity is representative of the interaction between the input variables. Further, the higher the first-order effect, the more important the variable is.

The interaction or combined effect of two variables X^i and X^j can also be calculated similarly. Alternatively, consider the expression:

$$\frac{V_{X^{-i}}(E_{X^i}(Y|X^{-i}))}{V(Y)};$$

this expression includes all interaction terms of all orders concerning all variables X^{-i}; any term involving X^i (both individual and any interaction with others) would not be included. As the sum of all the sensitivity indices must be equal to unity, the total effects (the sum of individual effects of X^i and all interactions with other quantities) can be calculated as:

$$S_T^i = 1 - \frac{V_{X^{-i}}(E_{X^i}(Y|X^{-i}))}{V(Y)} \tag{16}$$

The sum of the total effects indices of all variables is always greater than or equal to unity; equality holds when there is no interaction between the input quantities. (In this case, the first-order effects indices are equal to the total effects indices). If the total effects index is low, then it means that the input quantity is not important.

It is important to calculate both the first-order effects and the total effects indices. If the first-order index of a particular variable X^i is low, then it is not necessary that this variable is unimportant. The interaction of this variable with other variables may contribute significantly to the variance of Y and hence, there is a possibility that X^i is, in fact, an important variable. The effects of interaction are reflected in the total effects index. Further, the difference between the total effects index and the first-order effects index provides an estimate of the contribution of variance due to the interaction between X^i and other variables. Thus, both the first-order and total effects indices must be computed in order to assess the sensitivities of the variables.

V. Numerical Results

For the sake of illustration, this numerical example considers a particular aircraft (a Boeing 737-800) that takes off from a particular airport (Runway '1L-19R', in San Francisco, in this case), as shown in Fig. 2.

As mentioned before, the trajectory of an aircraft is described in terms of five different time-dependent quantities:

1. Airspeed (measured in knots)
2. Heading (angle, measured in degrees)
3. Altitude (measured in feet)

4. Latitude (measured in degrees)
5. Longtiude (measured in degrees)

A Latin Hypercube sampling approach with 1000 samples and 13 dimensions (each dimension corresponding to on uncertain quantity) is implemented in this paper, and the uncertainty in the entire trajectory, i.e., the uncertainty in each of the above 5 quantities is computed as a continuous function of time. The entire probability distribution for each of the five quantities is estimated for continuous future time-instants until $t = t_H$; this look-ahead time is chosen to be 5 minutes in this paper. Within this time-horizon, Y is predicted for every 10 seconds, and hence there are 31 predictions. Thus, the trajectory can be expressed as a matrix of 5 output quantities, 31 time-instants, and 1000 samples.

The lower and upper bounds (5% bounds, for the sake of illustration) of the five outputs along with the medians are plotted in Fig. 3 - Fig. 7, continuously as a function of time. For the sake of comparison, the actual/true trajectory of the flight under consideration is also indicated.

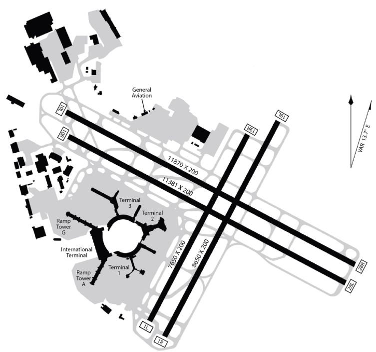


Figure 2: San Francisco Airport: Runways

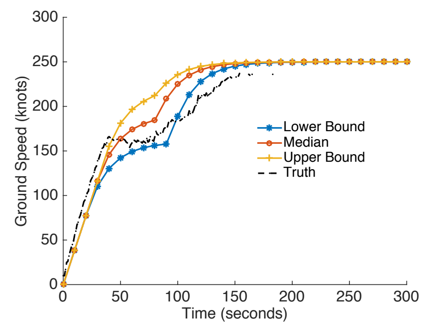


Figure 3: Trajectory: Groundspeed vs Time

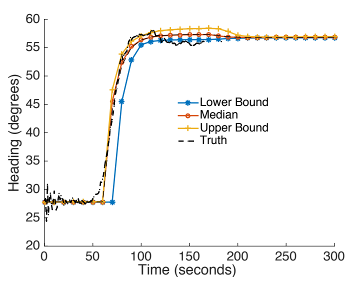


Figure 4: Trajectory: Heading vs Time

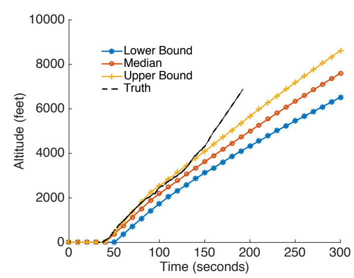


Figure 5: Trajectory: Altitude vs Time

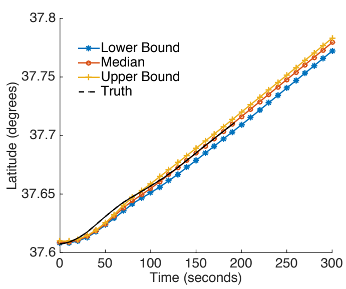


Figure 6: Trajectory: Latitude vs Time

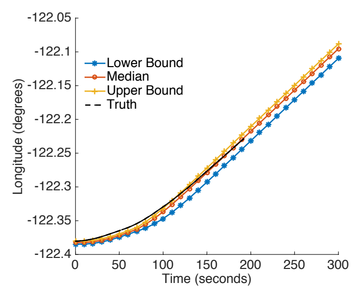


Figure 7: Trajectory: Longitude vs Time

From the results it can be seen that, the uncertainty in the prediction of latitude, longitude, and heading are reasonably small and remain so as a function of future time. The prediction of speed (Fig. 3) is initially uncertain, but the precision increases and the uncertainty decreases shortly thereafter. The most interesting aspect of the trajectory prediction results seems to be the prediction of altitude as a function of future time. It is easy to observe that the uncertainty in the altitude prediction (Fig. 5) steadily increases as a function of time.

Such an analysis, i.e., identifying which components of the trajectory are significantly affected because of uncertainty, is an important feature of the proposed approach; only the proposed systematic methodology can clearly arrive at such conclusions, i.e., compute significantly different uncertainty estimates for various quantities of interest within the trajectory-prediction framework. Such an analysis would have been impossible with existing approaches that simply impose uncertainty bounds after prediction is done.

Following uncertainty propagation analysis, global sensitivity analysis was also conducted by focusing the prediction of the probability distribution at the farthest time, i.e., at $t = t_H$ (chosen to be five minutes, for the sake of illustration). Since, the uncertainty in the altitude prediction is the most significant amongst the five predicted quantities, and global sensitivity analysis is performed in order to estimate the sensitivity of altitude to the various sources (13 uncertain variables, enumerated in Section III.A) of uncertainty, and the results are shown in Table 1.

Table 1: Sensitivity of Altitude Prediction

Uncertain Variable	Symbol	First-order Effects	Total Effects Index
Variable 1	$V_a(t_P)$	8.9×10^{-7}	1.5×10^{-6}
Variable 2	$V_z(t_P)$	3.3×10^{-5}	3.4×10^{-5}
Variable 3	$\chi_a(t_P)$	1.8×10^{-5}	5.0×10^{-3}
Variable 4	$h(t_P)$	2.8×10^{-5}	5.2×10^{-3}
Variable 5	$\lambda(t_P)$	2.6×10^{-6}	3.1×10^{-6}
Variable 6	$\tau(t_P)$	6.7×10^{-6}	7.4×10^{-6}
Variable 7	V_w	6.6×10^{-5}	6.7×10^{-5}
Variable 8	χ_w	1.4×10^{-5}	1.5×10^{-5}
Variable 9	V_h^*	2.2×10^{-6}	2.9×10^{-6}
Variable 10	V_a^*	8.8×10^{-1}	8.9×10^{-1}
Variable 11	G_z	7.4×10^{-2}	7.5×10^{-2}
Variable 12	G_a	6.1×10^{-3}	1.2×10^{-2}
Variable 13	J_χ	1.1×10^{-4}	1.2×10^{-4}

It is observed from Table 1, the "Variable 10", i.e, the commanded airspeed during the takeoff portion of the flight contributes to approximately 90% of the variance in altitude prediction. The second most important source of uncertainty arises from "Variable 11", i.e., a control parameter that affects how fast the aircraft responds to commanded changes in climb speed. This implies that, in order to reduce uncertainty in the altitude, it is necessary to reduce the uncertainty in the commanded airspeed. Such results from sensitivity analysis are also possible only within the proposed systematic uncertainty management framework, and this facilitates meaningful uncertainty reduction, which is important from the point of view of decision-making.

VI. Conclusion

This paper presented a computational approach for predicting the uncertainty in aircraft trajectories. This analysis is important for assessing the safety of the aircraft as well as the entire airspace. Using the uncertainty in the trajectory, it is possible to preemptively and proactively better predict impending aicraft separation issues, fuel shortage situations, airspace congestion, etc. in advance even before the occurrence of these events. The proposed approach is in stark contrast with existing approaches that are mostly reactive in nature, and hence will aid in the the next-generation air traffic analysis and management system.

A generic state-space modeling framework for trajectory prediction was presented and this framework was illustrated using a set of aircraft dynamic models. Several sources of uncertainties were investigated in this paper, and their joint effect on the overall uncertainty in the trajectory was quantified by estimating the probability distribution of the trajectory itself. In addition, global sensitivity analysis was conducted to quantify the contributions of the different sources of uncertainty to the overall uncertainty in the trajectory.

There are several possible directions for future work. As more complicated dynamic-models are included, the effect of additional sources of uncertainty can be quantified. While advanced Monte Carlo analysis methods have been shown to be suitable for uncertainty propagation, it is also worthwhile to investigate alternative approaches based on analytical methods[38,39] and surrogate modeling techniques.[40,41] Finally, it is also necessary to integrate the proposed techniques into airspace safety assessment methodologies and guide operational decision-making.

Acknowledgment

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